FYJC - MATHEMATICS & STATISTICS

PROBABILITY

PROBABILITY : CLASSICAL DEFINITION

01. a bag contains 7 white balls , 5 black balls , 4 red balls . If two balls are drawn at random from the bag , find the probability that one is red and other is black ans : 1/6

02. a box contains 4 white and 6 black balls . Two balls are drawn at random . Find the probability that both are of same color ans: 7/15

03. a box contains 8 red , 3 white and 9 blue balls . If 3 balls are drawn at random , find the probability that one of each color is drawn ans 18/95

04. A lot contains 12 items of which 4 are defective . Two items are drawn at random from the lot one after the other without replacement . Find the probability that both the items are non defective . ans : 14/33

05. Two cards are drawn at random from a pack of 52 playing cards . Find the probability that both are kings or both are queens ans : 2/221

06. a room has 3 lamps . From a collection of 10 light bulbs of which 6 are burnt out , a person selects 3 at random and puts them in the sockets . What is the probability that he will will have light from all three lamps ans :1/30

07. a room has three electric lamps . From a collection of 12 electric bulbs of which 6 are good , 3 bulbs are selected at random and put in the lamps . Find the probability that the room is lighted by at least one of the bulbs

ans:10/11

08. Six men in a company of 20 employees are graduates . If 3 men are picked out of 20 at random , what is the probability that they all are graduates ? What is the probability that at least one is a graduate

ans : 1/57 ; 194/285

09. A committee of 3 persons is to be formed from 3 company secretaries , 4 economists and 1 Chartered Accountant . What is the probability that

a) each of the three profession is represented in the committee ans : 3/14

b) the committee consists of Chartered accountant and atleast one economists ans : 9/28

10. From a group of 4 men , 3 women and 5 children , 4 persons are selected at random . Find the probability that the group selected consist of at least 2 women ans : 13/55

SUMS ON PERMUTATION

11. the letters of the word 'ORANGE' are arranged at random . Find the probability that vowels occupy odd places ans : 1 ⁄ 20

12. two adults and three children are sitting together on a sofa and watching cricket on TV . Find the probability that the adults are

a) occupying the corner seats ans : 1 ⁄ 10 b) sitting together ans : 2 ⁄ 5

13. 6 boys and 2 girls are to be seated for a photograph . Find the probability that the girls are not together

ans : 3 ⁄ 4

14. find the probability that in a random arrangement of the letters of the word 'TRIANGLE'

a) vowels are not together ans : 5 ⁄ 14

b) consonants occupy the odd places ans : 1 ⁄ 14

15. find the probability that in a random arrangement of the letters of the word 'LOGARITHM' , vowels occupy odd places ans : 5 ⁄ 42

16. Nine digits 1 , 2 , 3 ,......., 8 , 9 are arranged in a row to form a nine digit number . Find the probability that the digits 4 , 5 , 6 are together

a) in the order 654 ans : 1 ⁄ 72

b) in any order ans : 1 ⁄ 12

17. if a three digit number is formed out of 3, 5 , 6 , 8 with repetition of digits , find the probability that it is divisible by 5 ans : 1 ⁄ 3

18. Without repetition of digits , 4 digit numbers are formed using the digits 5 , 6 , 7 , 8 , 9 , 0 . Find the probability that the number formed is ODD and greater than 6000

19. Find the probability that a five digit number formed by using the digits 0 , 1, 2 , 3 , 4 and 5 is divisible by 3 without repeating the digits ans : 3 ⁄ 4

PROBABILITY : ADDITION THEOREM

01. Sixty percent of persons staying in a building read "Express" , fifty percent read "Times" ; while thirty percent read both . Find the probability that a randomly chosen person staying in the building reads at least one of the two ans : 0.8

02. A software company is bidding for computer programs A and B . Probability that the company will get program A is 3/5 , the probability that the company will get program B is 1/3 and probability that company will get both the programs is 1/ 8 . What is the probability that the company will get at least one program ans : 97/120

03. the probability that a person stopping at a petrol pump will ask for petrol is 0.80 , the probability that he will ask for water is 0.70 and the probability that he will ask for both is 0.65 . Find probability ; a person stopping at this petrol pump will ask for

a) either petrol or water ans : 0.85

b) neither petrol nor water ans : 0.15

04. a survey of families in a certain city showed that 60% of the families have a washing machine ; 55% have microwave oven ,40% of those who have a washing machine , have microwave oven . If a family is selected at random , find the probability that it has neither a washing machine nor microwave oven . ans : 0.09

05. the probability that a customer visiting a departmental store buys something from Medicines is 0.5 and that he buys something from Grocery is 0.4 and something from each of them is 0.25 . Find the probability that a visiting customer buys something from at least one of the two departments ans : 0.65

06. 100 students appeared for two examinations , 60 passed in first examination , 50 passed the second and 30 passed in both. Find the probability that a student selected at random

a) passed in at least one examination ans : 0.80

b) passed in exactly one examination ans : 0.50

c) failed in both the examination ans : 0.20

07. A factory employs both graduate and non graduate workers . The probability that a worker chosen at random is a graduate is 0.67 , that the worker is married is 0.72 and that the worker is a married graduate is 0.5 . Find probability that a worker chosen is

a) a graduate or married or both ans : 0.89

b) a married non – graduate ans : 0.22

08. probability that a contractor will get plumbing contract is 4/9 and he will not get an electrical contract is 1/3 . If the probability of getting at least one of the two contracts is 4/5 , then find the probability that he will get both the contracts ans :14/45

09. the probability that a student will get a gold medal is 0.4 and that he will not get a silver medal is 0.7 . If the probability of getting at least one medal is 0.6 , what is the probability that he will get

a) neither of the medals ans : 0.4 b) exactly one medal ans : 0.5

10. In a survey conducted by a Music club , it was observed that 45% people liked Indian classical music , while 50% liked western music and 15% liked neither Indian nor western music .If a individual is selected at random find the probability that he will like both the types of music ans : 0.10

11. Two dice are thrown together . What is the probability that sum of the numbers on two dice is 5 or number on the second die is greater that or equal to the number on the first die

ans : 23/36

12. 2 unbiased dice are rolled . Find the probability that the sum of the numbers on the uppermost faces is divisible by 2 or 4 ans : 1/2

13. if $P(A) = 1/4$; $P(B) = 2/5$; $P(A \cup B) = 1/2$, then find

a) $P(A \cap B)$ ans : $3/20$ b) $P(A \cap B')$ ans : $1/10$ c) $P(A' \cap B)$ ans : $1/4$

d) $P(A' \cap B')$ ans : $1/2$ e) $P(A' \cup B')$ ans : $17/20$ f) $P(A \cup B')$ ans : $3/4$

14. Events A , B and C form the partition of the sample space S

If $3P(A) = 2P(B) = P(C)$; find $P(A \cup B)$ ans : 5/11

PROBABILITY : MULTIPLICATION THEOREM

(INDEPENDENT EVENTS)

01. Probability that a student can solve a certain problem is 2/3 and that B can solve it is 1/3 . If both try independently , what is the probability that it is solved ans : 7/9

02. Probability that a student can solve a certain problem is 3/4 and that B can solve it is 4/5 . If both try independently , what is the probability that

a) the problem is solved ans : 19/20 b) the problem is not solved ans : 1/20

03. a problem is given to three students A ,B , C whose chances of solving it are 1/2 , 1/3 & 1/4 respectively . Find the probability that the problem will be solved ans : 3 ⁄ 4

04. The probability that a husband who is 55 years old living till he is 75 is 5 ⁄ 13 and his wife who is now 48 , living till she is 68 is 3⁄7 . Find the probability that

a) the couple will be alive 20 years hence ans : 15⁄91

b) at least one of them will be alive 20 years hence ans : 59⁄91

05. the probability that a man will be alive for 60 years is 3/5 and that his wife will be alive for 60 years is 2/3 . Find the probability that

a) both will be alive for 60 years b)only the man will be alive for 60 years

c) none will be alive for 60 years ans : 2 ⁄ 5 , 1⁄ 5 , 2 ⁄ 15

06. The probability that machines of a certain company require service in warranty period is 0.30 , while the probability that the dryers of the same company require service in warranty period is 0.10 . If the customer purchases both a machine and the dryer made by this company what is the probability that

a) both machine and dryer need warranty service ans : 0.03

b) neither machine nor dryer require warrant service ans : 0.63

07. Two students appear for an examination , their chances of passing the examination being 0.7 and 0.8 respectively . Find the probability that

a) at least one of them passes the examination ans : 0.94

b) only one of them passes the examination ans : 0.38

08. The probability that A can shoot a target is 3 ⁄ 4 and the probability that B can shoot is 3 ⁄ 5 . If A and B shoot independently of each other , find the probability that

a) the target is not shot at all ans : 1 ⁄ 10

b) exactly one of A and B shoot the target ans : 9 ⁄ 20

c) the target is shot ans : 9 ⁄ 10

09. if $P(A) = 3/5$; $P(B) = 2/3$; A and B are independent events

a) $P(A \cap B)$ **ans : 2 ⁄ 5** b) $P(A \cap B')$ **ans : 1 ⁄ 5** c) $P(A' \cap B)$ **ans : 4 ⁄ 15**

d) $P(A' \cap B')$ **ans : 2 ⁄ 15** e) $P(A \cup B)$ **ans : 13 ⁄ 15**

10. let A and B be events such that $P(A) = 0.3$ and $P(A \cup B) = 0.8$. if A and B are independent events , then find $P(B)$

PROBABILITY : CONDITIONAL PROBABILITY

(DEPENDENT EVENTS)

01. a purse contains 2 silver coins & 4 copper coins & second purse contains 4 silver coins & 3 copper coins . If a coin is selected from one of the purses , find probability that it is silver coin

ans : 19/42

02. there are two urns A and B . A contains 3 white & 5 red balls . B contains 2 white & 4 red balls . One urn is selected at random & a ball is drawn from it at random . Find the probability that the ball drawn is white ans : 17/48

03. An urn contains 7 red & 4 green balls . Another urn contains 4 red & 5 green balls . One urn is selected at random & a ball is drawn from it at random . Find the probability that it is a green ball ans : 91/198

04. a bag contains 3 red & 2 white balls . A second bag contains 2 red & 4 white balls . One ball is selected at random from the first bag and transferred to the second bag . Then a ball is drawn at random from the second bag . Find the probability that it is red ball ans : 13/35

05. a bag contains 5 white balls & 4 Black balls . A second bag contains 4 white balls & 6 black . One ball is selected at random from the first bag and transferred to the second bag . Then a ball is drawn at random from the second bag . Find the probability that it is black ball (58/99)

06. First urn contains 3 white and 4 black balls and second urn contains 5 white and 4 black balls . Two balls are transferred at random from the first urn and then one ball is drawn at random from the second urn . Find the probability that it is white .

07 a bag contains 10 white balls and 15 black balls . Two balls are drawn in succession without replacement . What is the probability that first is white and second is black . ans : 1/4

08. An urn contains 12 items of which 4 are defective . Two items are drawn at random from the urn one after the other without replacement . Find the probability that both items are non defective . ans : 14 / 33

09. Two cards are drawn from a pack of 52 cards one after the other without replacement . What is the probability that both the cards are ace ans : 1 / 221

10 A card is drawn from a well shuffled pack of 52 playing cards . It is kept aside . Then a second card is drawn from the remaining 51 cards . Find the probability that both the cards are queen

ans : 1 / 221

11 . two cards are drawn at random from a pack of 52 playing cards . find the probability that both are kings or both are queens ans : 2 / 221

12. in a class 40% students read Mathematics , 25% Biology and 15% both Mathematics and Biology . One student is selected at random The probability that he reads Mathematics if it is known that he reads Biology is ans : 3/5

13. in an examination 30% of the students have failed in mathematic , 20% of the students failed in chemistry and 10% have failed in both A student is selected at random . What is the probability that student has failed in mathematics if it is known that he has failed in chemistry ans : 1/2

14. At a certain examination out of 50 candidates 30 passed in Economics ; 35 passed in Psychology and 10 failed in both .A candidate is selected at random find the probability that he has passed in Economics , if it is known that he has passed in Psychology

ans : 5/7

15. Out of 50 members of a club , 40 like tea , 20 like coffee and 15 like both tea and coffee . A member is selected at random and it is found that he does not like coffee . Find the conditional probability that he likes tea ans : 5/6

16. a pair of dice is rolled . If the sum of the numbers appeared is 8 , find the probability that one die shows number 3 ans : 2/5

17. two fair dice are thrown . Find the probability that the sum of the points is at least 10 given that it exceeds 7 ans : 2/5

18. if $P(A) = 1/3$; $P(B) = 2/5$; $P(A \cup B) = 8/15$, then find

a) $P(A \mid B)$ ans : 1/2 b) $P(B \mid A)$ ans : 3/5

19. IF A and B are two events of the sample space S , such that

$P(A \cup B) = 5/6$; $P(A \cap B) = 1/3$; $P(B) = 1/3$.

Find

a) $P(A' \cap B')$ ans : 1/6 b) $P(B' \mid A)$ ans : 3/5

01. a bag contains 7 white balls , 5 black balls , 4 red balls . If two balls are drawn at random from the bag , find the probability that one is red and other is black

SOLUTION :

bag contains 7 white balls , 5 black balls , 4 red balls Total = 16 balls

exp : two balls are drawn at random from the bag , $n(S) = {}^{16}C_2$.

E ≡ one is red and other is black , $n(E) = {}^{4}C_1 \times {}^{5}C_1$

$$P(E) = \frac{n(E)}{n(S)} = \frac{{}^{4}C_1 \times {}^{5}C_1}{{}^{16}C_2} = \frac{4 \times 5}{120} = \frac{1}{6}$$

02. a box contains 4 white and 6 black balls . Two balls are drawn at random . Find the probability that both are of same color

SOLUTION :

bag contains 4 white balls & 6 black balls Total = 10 balls

exp : two balls are drawn at random from the bag , $n(S) = {}^{10}C_2$.

E ≡ both are of same color

(2 balls drawn are white OR 2 Balls drawn are black)

$n(E) = {}^{4}C_2 + {}^{6}C_2$

$$P(E) = \frac{n(E)}{n(S)} = \frac{{}^{4}C_2 + {}^{6}C_2}{{}^{10}C_2} = \frac{6 + 15}{45} = \frac{21}{45} = \frac{7}{15}$$

03. a box contains 8 red , 3 white and 9 blue balls . If 3 balls are drawn at random , find the probability that one of each color is drawn

SOLUTION :

bag contains 8 red , 3 white and 9 blue balls Total = 20 balls

exp : 3 balls are drawn at random from the bag , $n(S) = {}^{20}C_3$.

E ≡ one of each color is drawn

i.e. 1 red ball , 1 white ball and 1 blue ball is drawn

$n(E) = {}^{8}C_1 \times {}^{3}C_1 \times {}^{9}C_1$

$$P(E) = \frac{n(E)}{n(S)} = \frac{{}^{8}C_1 \times {}^{3}C_1 \times {}^{9}C_1}{{}^{20}C_3} = \frac{8 \times 3 \times 9}{1140} = \frac{18}{95}$$

04. A lot contains 12 items of which 4 are defective . Two items are drawn at random from the lot one after the other without replacement . Find the probability that both the items are non defective .

SOLUTION :

box contains 12 items (4 defective & 8 non defective)

exp : Two items are drawn at random from the lot , $n(S) = {}^{12}C_2$.

E ≡ both the items are non defective . $n(E) = {}^{8}C_2$

$$P(E) = \frac{n(E)}{n(S)} = \frac{{}^{8}C_2}{{}^{12}C_2} = \frac{28}{66} = \frac{14}{33}$$

05. Two cards are drawn at random from a pack of 52 playing cards . Find the probability that both are kings or both are queens

SOLUTION :

exp : Two cards are drawn at random from a pack of 52 playing cards ,

$n(S) = {}^{52}C_2$.

E ≡ both are kings or both are queens

$n(E) = {}^{4}C_2 + {}^{4}C_2$

$$P(E) = \frac{n(E)}{n(S)} = \frac{{}^{4}C_2 + {}^{4}C_2}{{}^{52}C_2} = \frac{6+6}{1326} = \frac{12}{1326} = \frac{2}{221}$$

06. a room has 3 lamps . From a collection of 10 light bulbs of which 6 are burnt out , a person selects 3 at random and puts them in the sockets . What is the probability that he will will have light from all three lamps

SOLUTION :

10 light bulbs (6 defective & 4 non defective)

exp : a person selects 3 bulbs at random

$n(S) = {}^{10}C_3$.

E ≡ person will have light from all 3 lamps

(i.e. person will have to draw 3 non defective bulbs)

$n(E) = {}^{4}C_3$

$$P(E) = \frac{n(E)}{n(S)} = \frac{{}^{4}C_3}{{}^{10}C_3} = \frac{4}{120} = \frac{1}{30}$$

07. a room has three electric lamps . From a collection of 12 electric bulbs of which 6 are good , 3 bulbs are selected at random and put in the lamps . Find the probability that the room is lighted by at least one of the bulbs

SOLUTION :

12 electric bulbs (6 good & 6 defective)

exp : a person selects 3 bulbs at random , $n(S) = {}^{12}C_3$.

E $\equiv$ the room is lighted by at least one of the bulbs

E' $\equiv$ the room is lighted by at none of the bulbs

(i.e. person will have to draw 3 defective bulbs)

$n(E) = {}^{6}C_3$

$$P(E') = \frac{n(E')}{n(S)} = \frac{{}^{6}C_3}{{}^{12}C_3} = \frac{20}{220} = \frac{1}{11}$$

$$P(E) = 1 - P(E') = 1 - \frac{1}{11} = \frac{10}{11}$$

08. Six men in a company of 20 employees are graduates . If 3 men are picked out of 20 at random , what is the probability that they all are graduates ? What is the probability that at least one is a graduate

SOLUTION :

20 employees (6 graduates & 14 non graduates)

exp : 3 men are picked at random , $n(S) = {}^{20}C_3$.

a) E $\equiv$ all are graduates , $n(E) = {}^{6}C_3$

$$P(E) = \frac{n(E)}{n(S)} = \frac{{}^{6}C_3}{{}^{20}C_3} = \frac{6.5.4.}{20.19.18} = \frac{1}{57}$$

b) E $\equiv$ at least one is a graduate

E' $\equiv$ none are graduates

(i.e. all 3 persons are non – graduates)

$n(E) = {}^{14}C_3$

$$P(E') = \frac{n(E')}{n(S)} = \frac{{}^{14}C_3}{{}^{20}C_3} = \frac{14 \times 13 \times 12}{20 \times 19 \times 18} = \frac{91}{285}$$

$$P(E) = 1 - P(E') = 1 - \frac{91}{285} = \frac{194}{285}$$

09. A committee of 3 persons is to be formed from 3 company secretaries , 4 economists and 1 Chartered Accountant . What is the probability that

a) each of the three profession is represented in the committee

b) the committee consists of Chartered accountant and atleast one economists

SOLUTION :

3 company secretaries , 4 economists and 1 Chartered Accountant Total = 8

exp : A committee of 3 persons is to be formed ; $n(S) = {}^{8}C_{3} = 56$.

a) E ≡ each of the three profession is represented in the committee

≡ committee will contain 1 CS , 1 economist and 1 Chartered Accountant

$n(E) = {}^{3}C_{1} \times {}^{4}C_{1} \times 1 = 12$

$$P(E) = \frac{n(E)}{n(S)} = \frac{12}{56} = \frac{3}{14}$$

b) E ≡ committee consists of Chartered accountant and atleast one economists

Case 1 : committee will contain 1 CA , 1 economist and 1 CS

this can be done in $1 \times {}^{4}C_{1} \times {}^{3}C_{1} = 12$ ways

Case 2 : committee will contain 1 CA & 2 economist

this can be done in $1 \times {}^{4}C_{2} = 6$ ways

By Fundamental principle of ADDITION : $n(E) = 12 + 6 = 18$

$$P(E) = \frac{n(E)}{n(S)} = \frac{18}{56} = \frac{9}{28}$$

10. From a group of 4 men , 3 women and 5 children , 4 persons are selected at random . Find the probability that the group selected consist of at least 2 women .

SOLUTION :

Exp : a group of 4 men , 3 women and 5 children , 4 persons are selected at random

$n(S) = {}^{12}C_{4} = 495$

E : the group selected consist of at least 2 women .

Case 1 : Group contains 2 women

2 women can be selected from 3 in ${}^{3}C_{2}$ ways . Having done that the remaining 2 can be selected from the remaining 9 in in ${}^{9}C_{2}$ ways .

By Fundamental principle of Multiplication ,

No of ways of forming the group = ${}^{3}C_{2} \times {}^{9}C_{2} = 108$

Case 2 : Group contains 3 women

3 women can be selected from 3 in $^{3}C_{3}$ ways . Having done that the remaining 1 person can be selected from the remaining 9 in in $^{9}C_{1}$ ways .

By Fundamental principle of Multiplication ,

No of ways of forming the group = $^{3}C_{3} \times {}^{9}C_{1}$ = 9

By Fundamental principle of ADDITION : n(E) = 108 + 9 = 117

$$P(E) = \frac{n(E)}{n(S)} = \frac{117}{495} = \frac{13}{55}$$

SUMS ON PERMUTATION

11. the letters of the word 'ORANGE' are arranged at random . Find the probability that vowels occupy odd places

SOLUTION :

Exp : letters of the word 'ORANGE' are arranged at random

n(S) = $^{6}P_{6}$ = 6!

Event : vowels occupy odd places

Letters 'O' , 'A' & 'E' can be arranged into the odd places 1 , 3 , & 5 in $^{3}P_{3}$ = 3! Ways

Having done that ;

The remaining 3 letters can be arranged into the remaining 3 places in $^{3}P_{3}$ = 3! Ways

Hence total ways = 3! x 3!

... BY FUNDAMENTAL PRINCIPLE OF MULTIPLICATION .

n(E) = 3! X 3!

$$P(E) = \frac{n(E)}{n(S)} = \frac{3! \times 3!}{6!} = \frac{3! \times \cancel{3!}}{6.5.4.\cancel{3!}} = \frac{3.2.1.}{6.5.4.} = \frac{1}{20}$$

12. two adults and three children are sitting together on a sofa and watching cricket on TV . Find the probability that the adults are

a) occupying the corner seats b) sitting together

SOLUTION : (a)

Exp : two adults and three children are to be arranged on a sofa

$n(S) = {}^5P_5 = 5!$

Event : adults are occupying the corner seats

Two Adults can be arranged into the corner seats in ${}^2P_2 = 2!$ Ways

Having done that ;

The three children can be arranged in ${}^3P_3 = 3!$ Ways

Hence total ways = 2! x 3!

... BY FUNDAMENTAL PRINCIPLE OF MULTIPLICATION .

$n(E) = 2! \times 3!$

$$P(E) = \frac{n(E)}{n(S)} = \frac{2! \times 3!}{5!} = \frac{2! \times \cancel{3!}}{5.\,4.\,\cancel{3!}} = \frac{2.1.}{5.4.} = \frac{1}{10}$$

SOLUTION : (b)

Event : adults are sitting together

Consider 2 adults as 1 set .

Hence 1 set of 2 adults and 3 children can be arranged amongst themselves in ${}^4P_4 = 4!$ Ways

Having done that ;

The two adults can be arranged in ${}^2P_2 = 2!$ Ways

Hence total ways = 4! x 2!

... BY FUNDAMENTAL PRINCIPLE OF MULTIPLICATION .

$n(E) = 4! \times 2!$

$$P(E) = \frac{n(E)}{n(S)} = \frac{4! \times 2!}{5!} = \frac{\cancel{4!} \times 2!}{5.\,\cancel{4!}} = \frac{2..}{5}$$

13. 6 boys and 2 girls are to be seated for a photograph . Find the probability that the girls are not together

SOLUTION : Exp : 6 boys and 2 girls are to be seated for a photograph

$n(S) = {}^8P_8 = 8!$

Event : girls are not together

$* \ B \ * \ B \ * \ B \ * \ B \ * \ B \ * \ B \ *$

The 2 girls can be arranged into any 2out of 7 places marked '∗' . in ${}^7P_2$ Ways

Having done that

The remaining 6 boys can be arranged in ${}^6P_6 = 6!$ Ways

Hence total ways $= {}^7P_2 \times 6!$

... BY FUNDAMENTAL PRINCIPLE OF MULTIPLICATION .

$n(E) = {}^7P_2 \text{ X } 6!$

$$P(E) = \frac{n(E)}{n(S)} = \frac{{}^7P_2 \text{ X } 6!}{8!} = \frac{{}^7P_2 \text{ X } \cancel{6!}}{8 .7. \cancel{6!}} = \frac{7.6}{8.7} = \frac{3}{4}$$

14. find the probability that in a random arrangement of the letters of the word 'TRIANGLE'

a) vowels are not together b) consonants occupy the odd places

SOLUTION : (a) Exp : letters of the word 'TRIANGLE' are arranged at random

$n(S) = {}^8P_8 = 8!$

Event : vowels are not together

$* \ C \ * \ C \ * \ C \ * \ C \ * \ C \ *$

the three vowels 'I' , 'A' & 'E' can be arranged into any three out of the 6 places marked '∗' in ${}^6P_3$ Ways

Having done that

The remaining 5 consonants can be arranged in ${}^5P_5 = 5!$ Ways

Hence total ways $= {}^6P_3 \times 5!$

... BY FUNDAMENTAL PRINCIPLE OF MULTIPLICATION .

$n(E) = {}^6P_3 \text{ X } 5!$

$$P(E) = \frac{n(E)}{n(S)} = \frac{{}^6P_3 \text{ X } 5!}{8!} = \frac{{}^6P_3 \text{ X } \cancel{5!}}{8.7.6.\cancel{5!}} = \frac{6.5.4.}{8.7.6.} = \frac{5}{14}$$

SOLUTION : (b)

Event : consonants occupy odd places

Odd places 1 . 3 . 5 . 7 can be filled by any 4 of the 5 consonants in ${}^5P_4$ ways

Having done that ;

The remaining 4 letters (3 vowels + 1 consonant) can be arranged into the remaining 4 places in ${}^4P_4 = 4!$ Ways

Hence total ways = ${}^5P_4 \times 4!$

... BY FUNDAMENTAL PRINCIPLE OF MULTIPLICATION .

$n(E) = {}^5P_4 \times 4!$

$$P(E) = \frac{n(E)}{n(S)} = \frac{{}^5P_4 \times 4!}{8!} = \frac{{}^5P_4 \times \cancel{4!}}{8.7.6.5.\cancel{4!}} = \frac{5.4.3.2}{8.76.5} = \frac{1}{14}$$

15. find the probability that in a random arrangement of the letters of the word 'LOGARITHM' , vowels occupy odd places

SOLUTION :

Exp : letters of the word 'LOGARITHM' are arranged at random

$n(S) = {}^9P_9 = 9!$

Event : vowels occupy odd places

Vowels O , A & 1 can be filled into any 3 out of the 5 odd places in ${}^5P_3$ ways

Having done that ;

The remaining 6 letters can be arranged into the remaining 6 places in ${}^6P_6 = 6!$ Ways

Hence total ways = ${}^5P_3 \times 6!$

... BY FUNDAMENTAL PRINCIPLE OF MULTIPLICATION .

$n(E) = {}^5P_3 \times 6!$

$$P(E) = \frac{n(E)}{n(S)} = \frac{{}^5P_3 \times 6!}{9!} = \frac{{}^5P_3 \times \cancel{6!}}{9.8.7.\cancel{6!}} = \frac{5.4.3.}{9.8.7} = \frac{5}{42}$$

16. Nine digits 1 , 2 , 3 ,……., 8 , 9 are arranged in a row to form a nine digit number . Find the probability that the digits 4 , 5 , 6 are together

a) in the order 654

b) in any order

Exp : digits 1 , 2 , 3 ,……., 8 , 9 are arranged in a row to form 9 digit number

$$n(S) = {}^{9}P_{9} = 9!$$

Solution (a)

Event : digits 4 , 5 , 6 are togther in the order 654

Consider digits 4, 5 , 6 as 1 set

Hence 1 set of digits 4 , 5 , 6 & remaining 6 digits can be arranged amongst themselves in ${}^{7}P_{7} = 7!$ Ways

Having done that ;

Since digits 4 , 5 , 6 are together in the order 654 they cannot be further arranged

Hence total arrangements = 7!

… BY FUNDAMENTAL PRINCIPLE OF MULTIPLICATION .

$$n(E) = 7!$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{7!}{9!} = \frac{\cancel{7!}}{9.8.\cancel{7!}} = \frac{1}{72}$$

Solution (b)

Event : digits 4 , 5 , 6 are together in any order

Consider digits 4, 5 , 6 as 1 set

Hence 1 set of digits 4 , 5 , 6 & remaining 6 digits can be arranged amongst themselves in ${}^{7}P_{7} = 7!$ Ways

Having done that ;

digits 4 , 5 , 6 can be further arranged in ${}^{3}P_{3} = 3!$ Ways

Hence total arrangements = 7! X 3!

… BY FUNDAMENTAL PRINCIPLE OF MULTIPLICATION .

$$n(E) = 7! \text{ X } 3!$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{7! \text{ X } 3!}{9!} = \frac{\cancel{7!} \text{ x } 6}{9.8.\cancel{7!}} = \frac{1}{12}$$

17. if a three digit number is formed out of 3, 5 , 6 , 8 with repetition of digits , find the probability that it is divisible by 5

Exp: a three digit number is formed out of 3, 5 , 6 , 8 with repetition of digits

Each of the three places can be filled by any of the 4 digits in 4 ways each

$n(S) = 4^3 = 64$

E : number formed is divisible by 5

Unit place can be filled by digit 5 in one way

Having done that , remaining two places can be filled by any of the 4 digits in 4 ways each , $n(E) = 1 \times 4 \times 4 = 16$

$P(E) = \frac{n(E)}{n(S)} = \frac{16}{48} = \frac{1}{3}$

18. Without repetition of digits , 4 digit numbers are formed using the digits 5 , 6 , 7 , 8 , 9 , 0 . Find the probability that the number formed is ODD and greater than 6000

Exp : 4 digit numbers to be formed using the digits 5 , 6 , 7 , 8 , 9 , 0 .

thousand place can be filled by any of the 5 digits (excluding 0) in ${}^5P_1$ ways

Remaining 3 places can be filled by any 3 of the remaining 5 digits in ${}^5P_3$ ways

By fundamental principle of multiplication , $n(S) = {}^5P_1 \times {}^5P_3 = 300$

E : number formed is ODD and greater than 6000

Case 1 : Thousand place is filled by digit 6 ,

Thousand place can be filled by digit 6 in 1 way

Since the number is ODD ,

Unit place can be filled by any one of the digits 5 , 7 , 9 in ${}^3P_1$ ways

Having done that ; remaining two places can be filled by any 2 of the remaining 4 digits in ${}^4P_2$ ways

By fundamental principle of multiplication ,

nos. formed $= 1 \times {}^3P_1 \times {}^4P_2 = 36$

Case 2 : Thousand place is filled by digit 7

Thousand place can be filled by digit '7' in 1 way

Since the number is ODD ,

Unit place can be filled by digit 5 or 9 in ${}^2P_1$ ways

Having done that ; remaining two places can be filled by any 2 of the remaining 4 digits in ${}^4P_2$ ways

By fundamental principle of multiplication ,

nos. formed $= {}^2P_1 \times {}^4P_2 = 24$

Case 3 : Thousand place is filled by digit 8 ,

Thousand place can be filled by digit 8 in 1 way

Since the number is ODD ,

Unit place can be filled by any one of the digits 5 , 7 , 9 in ${}^3P_1$ ways

Having done that ; remaining two places can be filled by any 2 of the remaining 4 digits in ${}^4P_2$ ways

By fundamental principle of multiplication ,

nos. formed = $1 \times {}^3P_1 \times {}^4P_2$ = 36

Case 4 : Thousand place is filled by digit 9

Thousand place can be filled by digit '9' in 1 way

Since the number is ODD ,

Unit place can be filled by digit 5 or 7 in ${}^2P_1$ ways

Having done that ; remaining two places can be filled by any 2 of the remaining 4 digits in ${}^4P_2$ ways

By fundamental principle of multiplication ,

nos. formed = ${}^2P_1 \times {}^4P_2$ = 24

Therefore ; By fundamental principle of ADDITION , n(E) = 36+24+36+24 = 120

$$P(E) = \frac{n(E)}{n(S)} = \frac{120}{300} = \frac{2}{5}$$

19. Find the probability that a five digit number formed by using the digits 0 , 1, 2 , 3 , 4 and 5 is divisible by 3 without repeating the digits

Exp. five digit number to be formed by using the digits 0 , 1, 2 , 3 , 4 and 5

Ten thousand place can be filled by any of the 5 digits (excluding 0) in ${}^5P_1$ ways

Remaining 4 places can be filled by any 4 of the remaining 5 digits in ${}^5P_4$ ways

By fundamental principle of multiplication , $n(S) = {}^5P_1 \times {}^5P_4 = 600$

E : number is divisible by 3

(NOTE : divisibility test for 3 is sum of digits have to be multiple of 3)

Case1 : 5 digit numbers formed using digits 0 , 1 , 2 , 4 , 5

Ten thousand place can be filled by any of the 4 digits (excluding 0) in ${}^4P_1$ ways

Remaining 4 places can be filled by any 4 of the remaining 4 digits in ${}^4P_4 = 4!$ ways

By fundamental principle of multiplication , nos. formed $= {}^4P_1 \times 4! = 96$

Case2 : 5 digit numbers formed using digits 1 , 2 , 3 , 4 , 5

5 places can be filled by the 5 digits in ${}^5P_5 = 5!$ ways

By fundamental principle of multiplication , nos. formed $= 5! = 120$

Therefore ; By fundamental principle of ADDITION , $n(E) = 96 + 120 = 216$

$$P(E) = \frac{n(E)}{n(S)} = \frac{216}{600} = \frac{9}{25}$$

01. Sixty percent of persons staying in a building read "Express" , fifty percent read "Times" ; while thirty percent read both . Find the probability that a randomly chosen person staying in the building reads at least one of the two

SOLUTION :

A : person reads 'EXPRESS' $P(A) = 60/100$

B : person reads 'TIMES' $P(B) = 50/100$

$A \cap B$: person reads 'BOTH' $P(A \cap B) = 30/100$

$E \equiv$ person reads at least one of the two

$E \equiv A \cup B$

$$P(E) = P(A \cup B)$$

$$= P(A) + P(B) - P(A \cap B)$$

$$= \frac{60}{100} + \frac{50}{100} - \frac{30}{100}$$

$$= \frac{80}{100}$$

$$= 0.8$$

02. A software company is bidding for computer programs A and B . Probability that the company will get program A is 3/5 , the probability that the company will get program B is 1/3 and probability that company will get both the programs is 1/8 . What is the probability that the company will get at least one program

SOLUTION :

A : company will get program A $P(A) = 3/5$

B : company will get program B $P(B) = 1/3$

$A \cap B$: company will get both the programs $P(A \cap B) = 1/8$

$E \equiv$ company will get at least one program

$E \equiv A \cup B$

$$P(E) = P(A \cup B)$$

$$= P(A) + P(B) - P(A \cap B)$$

$$= \frac{3}{5} + \frac{1}{3} - \frac{1}{8}$$

$$= \frac{72 + 40 - 15}{120}$$

$$= \frac{97}{120}$$

03. the probability that a person stopping at a petrol pump will ask for petrol is 0.80 , the probability that he will ask for water is 0.70 and the probability that he will ask for both is 0.65 . Find probability ; a person stopping at this petrol pump will ask for

a) either petrol or water b) neither petrol nor water

SOLUTION :

A	:	person will ask for 'PETROL'	P(A)	= 0.80
B	:	person will ask for 'WATER'	P(B)	= 0.70
$A \cap B$	:	person will ask for 'BOTH'	$P(A \cap B)$	= 0.65

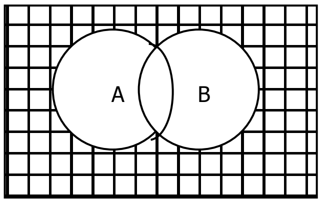
a) E ≡ person will ask for either petrol or water

$E \equiv A \cup B$

$$\begin{aligned} P(E) &= P(A \cup B) \\ &= P(A) + P(B) - P(A \cap B) \\ &= 0.80 + 0.70 - 0.65 \\ &= 0.85 \end{aligned}$$

b) E ≡ person will ask for neither petrol nor water

$E \equiv A' \cap B'$



$$\begin{aligned} P(E) &= P(A' \cap B') \\ &= 1 - P(A \cup B) \\ &= 1 - 0.85 \\ &= 0.15 \end{aligned}$$

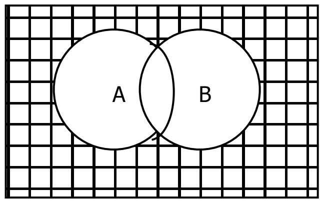
04. a survey of families in a certain city showed that 60% of the families have a washing machine ; 55% have microwave oven ,40% of those who have a washing machine , have microwave oven . If a family is selected at random , find the probability that it has neither a washing machine nor microwave oven . ans : 0.25

SOLUTION :

A	:	family has 'WASHING M/c'	P(A)	= $^{60}/_{100}$
B	:	family has 'MICROWAVE OVEN'	P(B)	= $^{55}/_{100}$
$A \cap B$	:	family has 'BOTH'	$P(A \cap B)$	= $^{40}/_{100} \times {}^{60}/_{100}$
				= $^{24}/_{100}$

$E \equiv$ family has neither a washing machine nor microwave oven .

$E \equiv A' \cap B'$



$$P(E) = P(A' \cap B')$$

$$= 1 - P(A \cup B)$$

$$= 1 - P(A) + P(B) - P(A \cap B)$$

$$= 1 - \left(\frac{60}{100} + \frac{55}{100} - \frac{24}{100} \right)$$

$$= 1 - \frac{91}{100}$$

$$= 9/100$$

05. the probability that a customer visiting a departmental store buys something from Medicines is 0.5 and that he buys something from Grocery is 0.4 and something from each of them is 0.25 . Find the probability that a visiting customer buys something from at least one of the two departments

SOLUTION :

A : customer will buy something from 'MEDICINES' $P(A) = 0.50$

B : customer will buy something from 'GROCERY' $P(B) = 0.40$

$A \cap B$: customer will buy something from 'EACH' $P(A \cap B) = 0.25$

$E \equiv$ customer will buy something from at least one of the two departments

$E \equiv A \cup B$

$$P(E) = P(A \cup B)$$

$$= P(A) + P(B) - P(A \cap B)$$

$$= 0.50 + 0.40 - 0.25$$

$$= 0.65$$

06. 100 students appeared for two examinations , 60 passed in first examination , 50 passed the second and 30 passed in both. Find the probability that a student selected at random

a) passed in at least one examination b) passed in exactly one examination

c) failed in both the examination

SOLUTION :

A : student has passed in 'FIRST' exam $P(A) = {}^{60}/_{100}$

B : student has passed in 'SECOND' exam $P(B) = {}^{50}/_{100}$

$A \cap B$: student has passed in 'BOTH' exams $P(A \cap B) = {}^{30}/_{100}$

a) E $\equiv$ student has passed in at least one examination

E $\equiv$ $A \cup B$

$$P(E) = P(A \cup B)$$

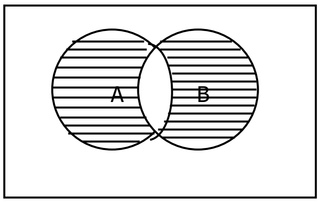
$$= P(A) + P(B) - P(A \cap B)$$

$$= \frac{60}{100} + \frac{50}{100} - \frac{30}{100}$$

$$= \frac{80}{100}$$

b) E $\equiv$ student has passed in exactly one examination

E $\equiv$ $(A \cup B) - (A \cap B)$



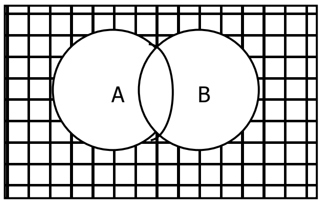
$$P(E) = P(A \cup B) - P(A \cap B)$$

$$= \frac{80}{100} - \frac{30}{100}$$

$$= 50/100$$

c) E $\equiv$ student failed in both the examination

E $\equiv$ $A' \cap B'$



$$P(E) = P(A' \cap B')$$

$$= 1 - P(A \cup B)$$

$$= 1 - 0.80$$

$$= 0.20$$

07. A factory employs both graduate and non graduate workers . The probability that a worker chosen at random is a graduate is 0.67 , that the worker is married is 0.72 and that the worker is a married graduate is 0.5 . Find probability that a worker chosen is

a) a graduate or married or both b) a married non – graduate

SOLUTION :

A	:	worker is a 'GRADUATE'	$P(A)$	= 0.67
B	:	worker is 'MARRIED'	$P(B)$	= 0.72
$A \cap B$	:	worker is a 'MARRIED GRADUATE'	$P(A \cap B)$	= 0.50

a) E $\equiv$ worker is a graduate or married or both

E $\equiv$ $A \cup B$

$$P(E) = P(A \cup B)$$

$$= P(A) + P(B) - P(A \cap B)$$

$$= 0.67 + 0.72 - 0.50$$

$$= 0.89$$

a) E $\equiv$ worker is a married non – graduate

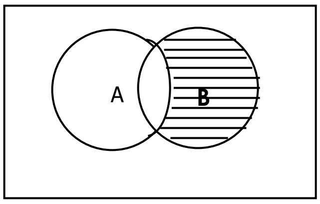
E $\equiv$ $A' \cap B$

$P(E) = P(A' \cap B)$

$= P(B) - P(A \cap B)$

$= 0.72 - 0.50$

$= 0.22$



08. probability that a contractor will get plumbing contract is 4/9 and he will not get an electrical contract is 1/3 . If the probability of getting at least one of the two contracts is 4/5 , then find the probability that he will get both the contracts

SOLUTION :

A : contractor will get 'PLUMBING CONTRACT' $P(A) = {}^4/_9$

B : contractor will get 'ELECTRICAL CONTRACT' $P(B) = 1 - {}^1/_3 = {}^2/_3$

$A \cup B$: contractor will get at least one contract $P(A \cup B) = {}^4/_5$

E $\equiv$ contractor will get both the contracts

E $\equiv$ $A \cap B$

$P(E) = P(A \cap B)$

$= P(A) + P(B) - P(A \cup B)$

$= \frac{4}{9} + \frac{2}{3} - \frac{4}{5}$

$= \frac{20 + 30 - 36}{45}$

$= \frac{14}{45}$

09. the probability that a student will get a gold medal is 0.4 and that he will not get a silver medal is 0.7 . If the probability of getting at least one medal is 0.6 , what is the probability that he will get a) neither of the medals b) exactly one medal

SOLUTION :

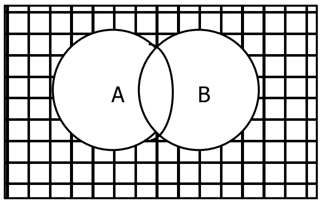
A : student will get 'GOLD' medal $P(A) = 0.4$

B : student will get 'SILVER' medal $P(B) = 1 - 0.7 = 0.3$

$A \cup B$: student will get at least one medal $P(A \cup B) = 0.6$

a) $E \equiv$ student will get neither of the medals

$E \equiv A' \cap B'$



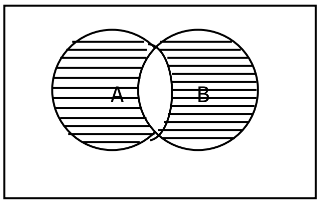
$P(E) = P(A' \cap B')$

$= 1 - P(A \cup B)$

$= 1 - 0.6 \quad = \; = 0.4$

b) $E \equiv$ student will get exactly one medal

$E \equiv (A \cup B) - (A \cap B)$



$P(E) = P(A \cup B) - P(A \cap B)$

$= P(A \cup B) - \left[P(A) + P(B) - P(A \cup B)\right]$

$= P(A \cup B) - P(A) - P(B) + P(A \cup B)$

$= 2P(A \cup B) - P(A) - P(B)$

$= 2(0.6) - 0.4 - 0.3 \qquad = 0.5$

10. In a survey conducted by a Music club , it was observed that 45% people liked Indian classical music , while 50% liked western music and 15% liked neither Indian nor western music .If a individual is selected at random find the probability that he will like both the types of music

SOLUTION :

A : individual likes 'INDIAN CLASSICAL' $P(A) = {}^{45}/_{100}$

B : individual likes 'WESTERN MUSIC' $P(B) = {}^{50}/_{100}$

$A' \cap B'$: individual likes neither 'INDIAN NOR WESTERN'

$P(A' \cap B') = {}^{15}/_{100}$

$1 - P(A \cup B) = {}^{15}/_{100}$

$P(A \cup B) = 1 - {}^{15}/_{100} = {}^{85}/_{100}$

$E \equiv$ student will like both the music

$E \equiv A \cap B$

$P(E) = P(A \cap B)$

$= P(A) + P(B) - P(A \cup B)$

$$= \frac{45}{100} + \frac{50}{100} - \frac{85}{100} \qquad = \frac{10}{100}$$

11. Two dice are thrown together . What is the probability that sum of the numbers on two dice is 5 or number on the second die is greater that or equal to the number on the first die

SOLUTION :

A : sum of numbers on the two dice is 5

$\equiv$ (1,4) , (2,3) , (3,2) , (4,1) $\quad$ n(A) = 4 $\quad \therefore$ P(A) = 4/36

B : number on the second die is greater than or equal to number on the first die

$\equiv$ (1,1) , (1,2) , (1,3) , (1,4) , (1,5) , (1,6) ,

(2,2) , (2,3) , (2,4) , (2,5) , (2,6) ,

(3,3) , (3,4) , (3,5) , (3,6) ,

(4,4) , (4,5) , (4,6) ,

(5,5) , (5,6) ,

(6,6) . $\quad$ n(B) = 21 $\quad \therefore$ P(B) = 21/36

$A \cap B$: sum of numbers on the two dice is 5 and number on the second die is greater than or equal to number on the first die

$\equiv$ (1,4) , (2,3) $\quad$ $n(A \cap B) = 2 \quad \therefore P(A \cap B) = 2/36$

$E \equiv A \cup B$

$P(E) = P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$$= \frac{4}{36} + \frac{21}{36} - \frac{2}{36}$$

$$= \frac{23}{36}$$

12. 2 unbiased dice are rolled . Find the probability that the sum of the numbers on the uppermost faces is divisible by 2 or 4

SOLUTION :

A $\equiv$ sum of numbers on the two dice is divisible by 2

$\equiv$ sum of numbers on the two dice is 2 , 4, 6 , 8 , 10 , 12

$\equiv$ { (1,1) , (1,3) , (2,2) , (3,1) , (1,5) , (2,4) , (3,3) , (4,2) , (5,1) , (2,6) , (3,5) , (4,4) , (5,3) , (6,2) , (4,6) , (5,5) , (6,4) , (6,6) }

n(A) = 18 , P(A) = 18/36

B $\equiv$ sum of numbers on the two dice is divisible by 4

$\equiv$ sum of numbers on the two dice is 4, 8 , 12

$\equiv$ { (1,3) , (2,2) , (3,1) , (2,6) , (3,5) , (4,4) , (5,3) , (6,2), (6,6) }

n(B) = 9 , P(B) = 9/36

$A \cap B$ $\equiv$ sum of numbers on the two dice is divisible by 2 & 4

$\equiv$ sum of numbers on the two dice is 4, 8 , 12

$\equiv$ { (1,3) , (2,2) , (3,1) , (2,6) , (3,5) , (4,4) , (5,3) , (6,2), (6,6) }

$n(A \cap B)$ = 9 , $P(A \cap B)$ = 9/36

$E \equiv A \cup B$

$P(E) = P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$= \dfrac{18}{36} + \dfrac{9}{36} - \dfrac{9}{36}$

$= \dfrac{1}{2}$

13. if $P(A) = 1/4$; $P(B) = 2/5$; $P(A \cup B) = 1/2$, then find

a) $P(A \cap B)$ b) $P(A \cap B')$ c) $P(A' \cap B)$ d) $P(A' \cap B')$ e) $P(A' \cup B')$

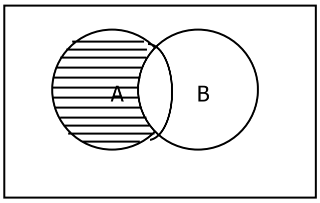
a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{1}{4} + \frac{2}{5} - \frac{1}{2}$$

$$= \frac{5 + 8 - 10}{20}$$

$$= \frac{3}{20}$$

b) $P(A \cap B') = P(A) - P(A \cap B)$

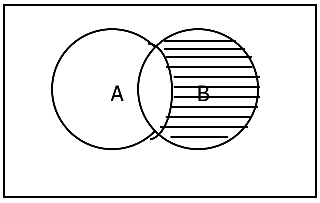


$$= \frac{1}{4} - \frac{3}{20}$$

$$= \frac{5 - 3}{20}$$

$$= \frac{2}{20} \quad = \frac{1}{10}$$

c) $P(A' \cap B) = P(B) - P(A \cap B)$

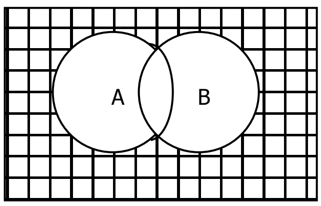


$$= \frac{2}{5} - \frac{3}{20}$$

$$= \frac{8 - 3}{20}$$

$$= \frac{5}{20} \quad = \frac{1}{4}$$

d) $P(A' \cap B') = 1 - P(A \cup B)$



$$= 1 - \frac{1}{2}$$

$$= \frac{1}{2}$$

e) $P(A' \cup B') = 1 - P(A \cap B)$

$$= 1 - \frac{3}{20}$$

$$= \frac{17}{20}$$

f) $P(A \cup B') = P(A) + P(B') - P(A \cap B')$

$$= \frac{1}{4} + \frac{3}{5} - \frac{1}{10}$$

$$= \frac{5 + 12 - 2}{20}$$

$$= 3/4$$

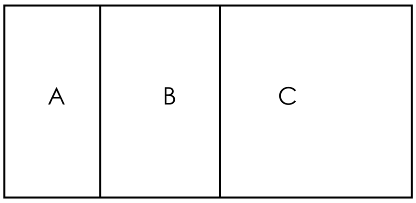
14. Events A , B and C form the partition of the sample space S

If $3P(A) = 2P(B) = P(C)$; find $P(A \cup B)$

$3P(A) = 2P(B) = P(C) = k$

$P(A) = k/3$, $P(B) = k/2$, $P(C) = k$

<u>A , B & C are MUTUALLY EXCLUSIVE & EXHAUSTIVE EVENTS</u>



$P(A) + P(B) + P(C) = 1$

$\frac{K}{3} + \frac{k}{2} + k = 1$

$\frac{2k + 3k + 6k}{6} = 1$

$\frac{11k}{6} = 1$

$k = \frac{6}{11}$

Hence : $P(A) = \frac{k}{3} = \frac{6/11}{3} = \frac{2}{11}$

$P(B) = \frac{k}{2} = \frac{6/11}{2} = \frac{3}{11}$

Now : $P(A \cup B) = P(A) + P(B)$

$= \frac{2}{11} + \frac{3}{11}$

$= \frac{5}{11}$

SOLUTION - SET

MULTIPLICATION THEOREM

(INDEPENDENT EVENTS)

01. Probability that a student A can solve a certain problem is 2/3 and that B can solve it is 1/3 . If both try independently , what is the probability that it is solved ans : 7/9

SOLUTION

A : student A can solve a problem $P(A) = {}^2/_3$, $P(A') = {}^1/_3$

B : student B can solve a problem $P(B) = {}^1/_3$, $P(B') = {}^2/_3$

$E \equiv$ problem is solved

$E' \equiv$ problem is not solved

$E' \equiv A' \cap B'$

$P(E') = P(A' \cap B')$

$$= P(A') \times P(B') = \frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$$

$$P(E) = 1 - P(E') = 1 - \frac{2}{9} = \frac{7}{9}$$

02. Probability that a student can solve a certain problem is 3/4 and that B can solve it is 4/5 . If both try independently , what is the probability that

a) the problem is solved

b) the problem is not solved

SOLUTION :

A : student A can solve a problem $P(A) = {}^3/_4$, $P(A') = {}^1/_4$

B : student B can solve a problem $P(B) = {}^4/_5$, $P(B') = {}^1/_5$

$E \equiv$ problem is solved

$E' \equiv$ problem is not solved

$E' \equiv A' \cap B'$

$P(E') = P(A' \cap B')$

$$= P(A') \times P(B') = \frac{1}{4} \times \frac{1}{5} = \frac{1}{20}$$

$$P(E) = 1 - P(E') = 1 - \frac{1}{20} = \frac{19}{20}$$

03. a problem is given to three students A ,B , C whose chances of solving it are 1/2 , 1/3 & 1/4 respectively . Find the probability that the problem will be solved ans : 3 ⁄ 4

SOLUTION :

A : student A can solve a problem $P(A) = {}^1/_2$, $P(A') = {}^1/_2$

B : student B can solve a problem $P(B) = {}^1/_3$, $P(B') = {}^2/_3$

C : student B can solve a problem $P(C) = {}^1/_4$, $P(C') = {}^3/_4$

E ≡ problem is solved

E' ≡ problem is not solved

$E' \equiv A' \cap B' \cap C'$

$$P(E') = P(A' \cap B' \cap C')$$

$$= P(A') \times P(B') \times P(C')$$

$$= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}$$

$$= \frac{1}{4}$$

$$P(E) = 1 - P(E')$$

$$= 1 - \frac{1}{4}$$

$$= \frac{3}{4}$$

04. The probability that a husband who is 55 years old living till he is 75 is 5 ⁄ 13 and his wife who is now 48 , living till she is 68 is 3⁄7 . Find the probability that

a) the couple will be alive 20 years hence

b) at least one of them will be alive 20 years hence

SOLUTION :

A : husband will be alive 20 years hence $P(A) = {}^5/_{13}$, $P(A') = {}^8/_{13}$

B : wife will be alive 20 years hence $P(B) = {}^3/_7$, $P(B') = {}^4/_7$

a) E ≡ couple will be alive 20 years hence

$E \equiv A \cap B$

$$P(E) = P(A \cap B)$$

$$= P(A) \times P(B)$$

$$= \frac{5}{13} \times \frac{3}{7}$$

$$= \frac{15}{91}$$

b) E $\equiv$ at least one of them will be alive 20 years hence

E' $\equiv$ none of them will be alive 20 years hence

E' $\equiv$ $A' \cap B'$

$P(E') = P(A' \cap B')$

$= P(A') \times P(B')$

$= \frac{8}{13} \times \frac{4}{7}$

$= \frac{32}{91}$

$P(E) = 1 - P(E')$

$= 1 - \frac{32}{91}$

$= \frac{59}{91}$

05. the probability that a man will be alive for 60 years is 3/5 and that his wife will be alive for 60 years is 2/3 . Find the probability that

a) both will be alive for 60 years

b) only the man will be alive for 60 years

c) none will be alive for 60 years

SOLUTION :

A : man will be alive for 60 years $P(A) = {}^{3}/_{5}$, $P(A') = {}^{2}/_{5}$

B : wife will be alive for 60 years $P(B) = {}^{2}/_{3}$, $P(B') = {}^{1}/_{3}$

a) E $\equiv$ both will be alive for 60 years

E $\equiv$ $A \cap B$

$P(E) = P(A \cap B)$

$= P(A) \times P(B)$

$= \frac{3}{5} \times \frac{2}{3}$

$= \frac{2}{5}$

b) E $\equiv$ only the man will be alive for 60 years

E $\equiv$ $A \cap B'$

$P(E) = P(A \cap B')$

$= P(A) \times P(B')$

$= \frac{3}{5} \times \frac{1}{3}$

$= \frac{1}{5}$

c) $E \equiv$ none will be alive for 60 years

$E \equiv A' \cap B'$

$P(E) = P(A' \cap B')$

$= P(A') \times P(B')$

$= \dfrac{2}{5} \times \dfrac{1}{3}$

$= \dfrac{2}{15}$

06. The probability that machines of a certain company require service in warranty period is 0.30 , while the probability that the dryers of the same company require service in warranty period is 0.10 . If the customer purchases both a machine and the dryer made by this company what is the probability that

a) both machine and dryer need warranty service ans : 0.03

b) neither machine nor dryer require warrant service ans : 0.63

SOLUTION :

A : machine requires service in warranty period $P(A) = 0.30$, $P(A') = 0.70$

B : dryer requires service in warranty period $P(B) = 0.10$, $P(B') = 0.90$

a) $E \equiv$ both machine and dryer need warranty service

$E \equiv A \cap B$

$P(E) = P(A \cap B)$

$= P(A) \times P(B)$

$= 0.3 \times 0.1$

$= 0.03$

b) $E \equiv$ neither machine nor dryer require warrant service

$E \equiv A' \cap B'$

$P(E) = P(A' \cap B')$

$= P(A') \times P(B')$

$= 0.7 \times 0.9$

$= 0.63$

07. Two students appear for an examination , their chances of passing the examination being 0.7 and 0.8 respectively . Find the probability that

a) at least one of them passes the examination ans : 0.94

b) only one of them passes the examination ans : 0.38

SOLUTION :

A : student A passes the examination $P(A) = 0.7$, $P(A') = 0.3$

B : student B passes the examination $P(B) = 0.8$, $P(B') = 0.2$

a) $E \equiv$ at least one of them passes the examination

$E' \equiv$ none of them passes the examination

$E' \equiv A' \cap B'$

$P(E') = P(A' \cap B')$

$= P(A') \times P(B')$

$= 0.3 \times 0.2$

$= 0.06$

$P(E) = 1 - P(E')$

$= 1 - 0.06 = 0.94$

b) $E \equiv$ only one of them passes the examination

$E \equiv E_1 \cup E_2$

Where

$E_1 \equiv$ A passes and B fails

$E_1 \equiv A \cap B'$

$P(E_1) = P(A \cap B')$

$= P(A) \times P(B')$

$= 0.7 \times 0.2 = 0.14$

$E_2 \equiv$ A fails and B passes

$E_2 \equiv A' \cap B$

$P(E_1) = P(A' \cap B)$

$= P(A') \times P(B)$

$= 0.3 \times 0.8 = 0.24$

Now

$E \equiv E_1 \cup E_2$

$P(E) = P(E_1 \cup E_2)$

$= P(E1) + P(E2)$

....E1 & E2 are mutually exlusive

$= 0.14 + 0.24$

$= 0.38$

08. The probability that A can shoot a target is 3 ⁄ 4 and the probability that B can shoot is 3 ⁄ 5 . If A and B shoot independently of each other , find the probability that

a) the target is not shot at all — ans : 1 ⁄ 10

b) exactly one of A and B shoot the target — ans : 9 ⁄ 20

c) the target is shot — ans : 9 ⁄ 10

SOLUTION :

A : A will shoot the target $P(A) = 3/4$, $P(A') = 1/4$

B : B will shoot the target $P(B) = 3/5$, $P(B') = 2/5$

c) $E \equiv$ target is shot

$E' \equiv$ target is not shot at all

$E' \equiv A' \cap B'$

$P(E') = P(A' \cap B')$

$$= P(A') \times P(B') = \frac{1}{4} \times \frac{2}{5} = \frac{1}{10}$$

$$P(E) = 1 - P(E') = 1 - \frac{1}{10} = \frac{9}{10}$$

b) $E \equiv$ exactly one of A and B shoot the target

$E \equiv E_1 \cup E_2$

Where

$E_1 \equiv$ A shoots and B fails

$E_1 \equiv A \cap B'$

$P(E_1) = P(A \cap B')$

$$= P(A) \times P(B') = 3/4 \times 2/5 = 6/20$$

$E_2 \equiv$ A fails and B shoots

$E_2 \equiv A' \cap B$

$P(E_1) = P(A' \cap B)$

$$= P(A') \times P(B) = 1/4 \times 3/5 = 3/20$$

Now

$E \equiv E_1 \cup E_2$

$P(E) = P(E_1 \cup E_2)$

$= P(E_1) + P(E_2)$

....E1 & E2 are mutually exlusive

$= 6/20 + 3/20$

$= 9/20$

10. let A and B be events such that $P(A) = 0.3$ and $P(A \cup B) = 0.8$. if A and B are independent events , then find $P(B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.8 = 0.3 + P(B) - P(A).P(B)$ ..... A and B are independent events

$0.5 = P(B) - 0.3P(B)$

$0.5 = 0.7P(B)$

$P(B) = 5/7$

01. a purse contains 2 silver coins & 4 copper coins & second purse contains 4 silver coins & 3 copper coins . If a coin is selected from one of the purses , find probability that it is silver coin

Purse 1 (6 coins)	Purse 2 (7 coins)
2 silver , 4 copper	4 silver , 3 copper

exp : a coin is selected from one of the purses

$E \equiv$ coin is a silver coin

$E \equiv E_1 \cup E_2$

Where

$E_1 \equiv$ purse 1 is selected AND a silver coin is drawn from it

$E_1 \equiv A \cap B$

$P(E_1) = P(A \cap B)$

$P(E_1) = P(A) \times P(B \mid A)$

$= \frac{1}{2} \times \frac{2}{6}$

$= \frac{1}{6}$

$E_2 \equiv$ purse 2 is selected AND a silver coin is drawn from it

$E_2 \equiv A \cap B$

$P(E_2) = P(A \cap B)$

$P(E_2) = P(A) \times P(B \mid A)$

$= \frac{1}{2} \times \frac{4}{7}$

$= \frac{2}{7}$

Now ;

$E \equiv E_1 \cup E_2$

$P(E) = P(E_1 \cup E_2)$

$P(E) = P(E_1) + P(E_2)$ …….. E_1 & E_2 are MUTUALLY EXCLUSIVE

$= \frac{1}{6} + \frac{2}{7}$

$= \frac{19}{42}$

02. there are two urns A and B . A contains 3 white & 5 red balls . B contains 2 white & 4 red balls . One urn is selected at random & a ball is drawn from it at random . Find the probability that the ball drawn is white

URN A (8 Balls)	URN B (6 Balls)
3 white , 5 red	2 white , 4 red

exp : One urn is selected at random & a ball is drawn from it at random

$E \equiv$ ball drawn is WHITE

$E \equiv E_1 \cup E_2$

Where

$E_1 \equiv$ URN A is selected AND a white ball is drawn from it

$E_1 \equiv A \cap B$

$P(E_1) = P(A \cap B)$

$P(E_1) = P(A) \times P(B \mid A)$

$$= \frac{1}{2} \times \frac{3}{8}$$

$$= \frac{3}{16}$$

$E_2 \equiv$ URN B is selected AND a white ball is drawn from it

$E_2 \equiv A \cap B$

$P(E_2) = P(A \cap B)$

$P(E_2) = P(A) \times P(B \mid A)$

$$= \frac{1}{2} \times \frac{2}{6}$$

$$= \frac{1}{6}$$

Now ;

$E \equiv E_1 \cup E_2$

$P(E) = P(E_1 \cup E_2)$

$P(E) = P(E_1) + P(E_2)$ …….. E_1 & E_2 are MUTUALLY EXCLUSIVE

$$= \frac{3}{16} + \frac{1}{6}$$

$$= \frac{34}{96} \qquad = \frac{17}{48}$$

03. An urn contains 7 red & 4 green balls . Another urn contains 4 red & 5 green balls . One urn is selected at random & a ball is drawn from it at random . Find the probability that it is a green ball

URN 1 (11 Balls)	URN 2 (9 Balls)
7 red , 4 green	4 red, 5 green

exp : One urn is selected at random & a ball is drawn from it at random

$E \equiv$ ball drawn is GREEN

$E \equiv E_1 \cup E_2$

Where

$E_1 \equiv$ URN 1 is selected AND a green ball is drawn from it

$E_1 \equiv A \cap B$

$P(E_1) = P(A \cap B)$

$P(E_1) = P(A) \times P(B \mid A)$

$= \frac{1}{2} \times \frac{4}{11}$

$= \frac{2}{11}$

$E_2 \equiv$ URN 2 is selected AND a green ball is drawn from it

$E_2 \equiv A \cap B$

$P(E_2) = P(A \cap B)$

$P(E_2) = P(A) \times P(B \mid A)$

$= \frac{1}{2} \times \frac{5}{9}$

$= \frac{5}{18}$

Now ;

$E \equiv E_1 \cup E_2$

$P(E) = P(E_1 \cup E_2)$

$P(E) = P(E_1) + P(E_2)$ …….. E_1 & E_2 are MUTUALLY EXCLUSIVE

$= \frac{2}{11} + \frac{5}{18}$

$= \frac{91}{198}$

04. An urn contains 7 red & 4 green balls . Another urn contains 4 red & 5 green balls . One urn is selected at random & a ball is drawn from it at random . Find the probability that it is a green ball

URN 1 (11 Balls)	URN 2 (9 Balls)
5 silver , 3 copper	3 silver, 4 copper

exp : One urn is selected at random & 2 coins are drawn from it

$E \equiv$ both the coins are Silver

$E \equiv E_1 \cup E_2$

Where

$E_1 \equiv$ URN 1 is selected AND silver coins are drawn from it

$E_1 \equiv A \cap B$

$P(E_1) = P(A \cap B)$

$P(E_1) = P(A) \times P(B \mid A)$

$$= \frac{1}{2} \times \frac{{}^{5}C_2}{{}^{8}C_2}$$

$$= \frac{1}{2} \times \frac{5.4}{8.7} = \frac{5}{28}$$

$E_2 \equiv$ URN 2 is selected AND silver

$E_2 \equiv A \cap B$

$P(E_2) = P(A \cap B)$

$P(E_2) = P(A) \times P(B \mid A)$

$$= \frac{1}{2} \times \frac{{}^{3}C_2}{{}^{7}C_2}$$

$$= \frac{1}{2} \times \frac{3.2}{7.6} = \frac{1}{14}$$

Now ;

$E \equiv E_1 \cup E_2$

$P(E) = P(E_1 \cup E_2)$

$P(E) = P(E_1) + P(E_2)$ .... E_1 & E_2 are MUTUALLY EXCLUSIVE

$$= \frac{5}{28} + \frac{1}{14}$$

$$= \frac{7}{28} = \frac{1}{4}$$

05. a bag contains 3 red & 2 white balls . A second bag contains 2 red & 4 white balls . One ball is selected at random from the first bag and transferred to the second bag . Then a ball is drawn at random from the second bag . Find the probability that it is red ball

BAG 1 (5 Balls)	BAG 2 (6 Balls)
3 red , 2 white	2 red, 4 white

exp : One ball is selected at random from the first bag and transferred to the second bag . Then a ball is drawn at random from the second bag

E $\equiv$ ball drawn is RED

E $\equiv$ $E_1 \cup E_2$

Where

$E_1 \equiv$ Red ball is transferred from Bag 1 to Bag 2 AND a Red ball is drawn from bag 2

$E_1 \equiv A \cap B$

$P(E_1) = P(A \cap B)$

$P(E_1) = P(A) \times P(B \mid A)$

$= \frac{3}{5} \times \frac{3}{7}$

NEW CONFIGURATION OF BAG 2

3 RED ; 4 WHITE

$= \frac{9}{35}$

$E_2 \equiv$ White ball is transferred from Bag 1 to Bag 2 AND a Red ball is drawn from bag 2

$E_2 \equiv A \cap B$

$P(E_2) = P(A \cap B)$

$P(E_2) = P(A) \times P(B \mid A)$

$= \frac{2}{5} \times \frac{2}{7}$

NEW CONFIGURATION OF BAG 2

2 RED ; 5 WHITE

$= \frac{4}{35}$

Now ;

$E \equiv E_1 \cup E_2$

$P(E) = P(E_1 \cup E_2)$

$P(E) = P(E_1) + P(E_2)$ …….. E_1 & E_2 are MUTUALLY EXCLUSIVE

$= \frac{9}{35} + \frac{4}{35}$

$= 13/35$

06. a bag contains 5 white balls & 4 Black balls . A second bag contains 4 white balls & 6 black . One ball is selected at random from the first bag and transferred to the second bag . Then a ball is drawn at random from the second bag . Find the probability that it is black ball

BAG 1 (9 Balls)	BAG 2 (10 Balls)
5 white , 4 Black	4 white, 6 black

exp : One ball is selected at random from the first bag and transferred to the second bag . Then a ball is drawn at random from the second bag

$E \equiv$ ball drawn is BLACK

$E \equiv E_1 \cup E_2$

Where

$E_1 \equiv$ White ball is transferred from Bag 1 to Bag 2 AND a Black ball is drawn from bag 2

$E_1 \equiv A \cap B$

$P(E_1) = P(A \cap B)$

$P(E_1) = P(A) \times P(B\,|\,A)$

$= \dfrac{5}{9} \times \dfrac{6}{11}$

NEW CONFIGURATION OF BAG 2

5 white, 6 black

$= \dfrac{30}{99}$

$E_2 \equiv$ Black ball is transferred from Bag 1 to Bag 2 AND a Black ball is drawn from bag 2

$E_2 \equiv A \cap B$

$P(E_2) = P(A \cap B)$

$P(E_2) = P(A) \times P(B\,|\,A)$

$= \dfrac{4}{9} \times \dfrac{7}{11}$

NEW CONFIGURATION OF BAG 2

4 white, 7 black

$= \dfrac{28}{99}$

Now ;

$E \equiv E_1 \cup E_2$

$P(E) = P(E_1 \cup E_2)$

$P(E) = P(E_1) + P(E_2)$ …….. E_1 & E_2 are MUTUALLY EXCLUSIVE

$= \dfrac{30}{99} + \dfrac{28}{99}$

$= 58/99$

07. First urn contains 3 white and 4 black balls and second urn contains 5 white and 4 black balls . Two balls are transferred at random from the first urn and then one ball is drawn at random from the second urn . Find the probability that it is white .

URN 1 (7 Balls)	URN 2 (9 Balls)
3 white , 4 Black	5 white, 4 black

exp : Two balls are transferred at random from the first urn and then one ball is drawn at random from the second urn

E $\equiv$ ball drawn is WHITE

E $\equiv$ $E_1 \cup E_2 \cup E_3$

Where

$E_1 \equiv$ 2 White balls are transferred from Urn 1 to Urn 2 AND a White ball is drawn from Urn 2

$E_1 \equiv A \cap B$

$P(E_1) = P(A \cap B)$

$P(E_1) = P(A) \times P(B | A)$

$= \dfrac{{}^3C_2}{{}^7C_2} \times \dfrac{7}{11}$

NEW CONFIGURATION OF BAG 2

7 white, 4 black

$= \dfrac{3}{21} \times \dfrac{7}{11}$

$= \dfrac{1}{11}$

$E_2 \equiv$ 2 Black balls are transferred from Urn 1 to Urn 2 AND a White ball is drawn from urn 2

$E_2 \equiv A \cap B$

$P(E_2) = P(A \cap B)$

$P(E_2) = P(A) \times P(B | A)$

$= \dfrac{{}^4C_2}{{}^7C_2} \times \dfrac{7}{11}$

NEW CONFIGURATION OF BAG 2

5 white, 6 black

$= \dfrac{6}{21} \times \dfrac{5}{11}$

$= \dfrac{10}{77}$

$E_3 \equiv$ 1 white & 1 black ball is transferred AND a White ball is drawn from urn 2

$E_3 \equiv A \cap B$

$P(E_3) = P(A \cap B)$

$P(E_3) = P(A) \times P(B | A)$

$= \dfrac{{}^3C_1 \times {}^4C_1}{{}^7C_2} \times \dfrac{6}{11}$

NEW CONFIGURATION OF BAG 2

6 white, 5 black

$= \dfrac{12}{21} \times \dfrac{6}{11}$

$= \dfrac{24}{77}$

Now ;

$E \equiv E_1 \cup E_2 \cup E_3$

$P(E) = P(E_1 \cup E_2 \cup E_3)$

$P(E) = P(E_1) + P(E_2) + P(E_2)$ .... E_1 , E_2 & E_3 are MUTUALLY EXCLUSIVE

$= \dfrac{1}{11} + \dfrac{10}{77} + \dfrac{24}{77}$

$= \dfrac{7 + 10 + 24}{77}$

$= \dfrac{41}{77}$

08. a bag contains 10 white balls and 15 black balls . Two balls are drawn in succession without replacement . What is the probability that first is white and second is black .

exp : Two balls are drawn in succession without replacement

$E \equiv$ first is white AND second is black

$E \equiv A \cap B$

$P(E) = P(A \cap B)$

$P(E) = P(A) \times P(B | A)$

$= \dfrac{10}{25} \times \dfrac{15}{24}$

$= \dfrac{1}{4}$

09. An urn contains 12 items of which 4 are defective . Two items are drawn at random from the urn one after the other without replacement . Find the probability that both items are non defective .

12 items - 4 defective & 8 non defective

exp : Two items are drawn at random from the urn one after the other without replacement

E $\equiv$ both items are non defective .

E $\equiv$ First item is non - defective AND second item is non - defective

E $\equiv$ A $\cap$ B

$P(E) = P(A \cap B)$

$P(E) = P(A) \times P(B \mid A)$

$= \frac{8}{12} \times \frac{7}{11}$

$= \frac{14}{33}$

10. Two cards are drawn from a pack of 52 cards one after the other without replacement . What is the probability that both the cards are ace

exp : Two cards are drawn from a pack of 52 cards one after the other without replacement

E $\equiv$ both the cards are ace

E $\equiv$ first card is an ace AND second card is an ace

E $\equiv$ A $\cap$ B

$P(E) = P(A \cap B)$

$P(E) = P(A) \times P(B \mid A)$

$= \frac{4}{52} \times \frac{3}{51}$

FOR Q 10 : REFER THE SAME SOLUTION

$= \frac{1}{221}$

12 two cards are drawn at random from a pack of 52 playing cards . find the probability that both are kings or both are queens

exp : Two cards are drawn from a pack of 52 cards one after the other without replacement

$E \equiv$ both are Kings OR both are Queens

$E \equiv E_1 \cup E_2$

Where

$E_1 \equiv$ first card is a King AND second card is a King

$E_1 \equiv A \cap B$

$P(E_1) = P(A \cap B)$

$P(E_1) = P(A) \times P(B \mid A)$

$= \dfrac{4}{52} \times \dfrac{3}{51}$

$= \dfrac{1}{221}$

$E_2 \equiv$ first card is a Queen AND second card is a Queen

$E_2 \equiv A \cap B$

$P(E_2) = P(A \cap B)$

$P(E_2) = P(A) \times P(B \mid A)$

$= \dfrac{4}{53} \times \dfrac{3}{51}$

$= \dfrac{1}{221}$

Now ;

$E \equiv E_1 \cup E_2$

$P(E) = P(E_1 \cup E_2)$

$P(E) = P(E_1) + P(E_2)$ …….. E_1 & E_2 are MUTUALLY EXCLUSIVE

$= \dfrac{1}{221} + \dfrac{1}{221}$

$= 2/221$

13. in a class 40% students read Mathematics , 25% Biology and 15% both Mathematics and Biology . One student is selected at random The probability that he reads Mathematics if it is known that he reads Biology is

SOLUTION :

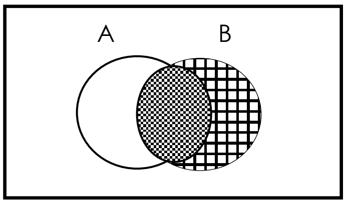
A : student reads 'Mathematics' $P(A) = {}^{40}/_{100}$

B : student reads 'Biology' $P(B) = {}^{25}/_{100}$

$A \cap B$: student reads 'BOTH' $P(A \cap B) = {}^{15}/_{100}$

$E \equiv$ he reads Mathematics if it is known that he reads Biology

$E \equiv A | B$



$$P(E) = P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{{}^{15}/_{100}}{{}^{25}/_{100}} = \frac{3}{5}$$

14. in an examination 30% of the students have failed in mathematic , 20% of the students failed in chemistry and 10% have failed in both A student is selected at random . What is the probability that student has failed in mathematics if it is known that he has failed in chemistry

SOLUTION :

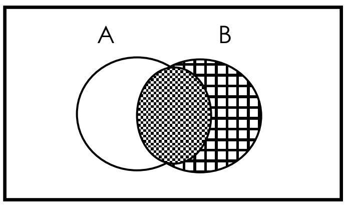
A : student failed in 'Mathematics' $P(A) = {}^{30}/_{100}$

B : student failed in 'Chemistry' $P(B) = {}^{20}/_{100}$

$A \cap B$: student failed in 'BOTH' $P(A \cap B) = {}^{10}/_{100}$

$E \equiv$ student has failed in mathematics if it is known that he has failed in chemistry

$E \equiv A | B$



$$P(E) = P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{{}^{10}/_{100}}{{}^{20}/_{100}} = \frac{1}{2}$$

15. At a certain examination out of 50 candidates 30 passed in Economics ; 35 passed in Psychology and 10 failed in both .A candidate is selected at random find the probability that he has passed in Economics , if it is known that he has passed in Psychology

SOLUTION :

A : candidate has passed in 'Economics' $P(A) = {}^{30}/_{50}$

B : candidate has passed in 'Psychology' $P(B) = {}^{35}/_{50}$

$A' \cap B'$: candidate has failed in 'BOTH' $P(A' \cap B') = {}^{10}/_{50}$

Now , $P(A' \cap B') = 1 - P(A \cup B)$

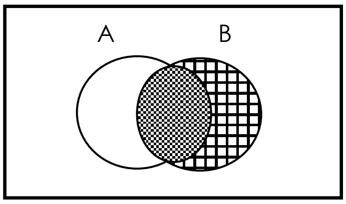
$\therefore \quad 10/50 = 1 - P(A \cup B)$

$\therefore \quad P(A \cup B) = 1 - {}^{10}/_{50}$

$= {}^{40}/_{50}$

E $\equiv$ candidate has passed in Economics , if it is known that he has passed in Psychology

E $\equiv$ A | B



$P(E) = P(A \mid B)$

$= \dfrac{P(A \cap B)}{P(B)}$

$= \dfrac{P(A) + P(B) - P(A \cup B)}{P(B)}$

$= \dfrac{{}^{30}/_{50} + {}^{35}/_{50} - {}^{40}/_{50}.}{{}^{35}/_{50}}$

$= {}^{25}/_{35}$

$= {}^{5}/_{7}$

16. Out of 50 members of a club , 40 like tea , 20 like coffee and 15 like both tea and coffee . A member is selected at random and it is found that he does not like coffee . Find the conditional probability that he likes tea

SOLUTION :

A : member likes 'TEA' $P(A) = {}^{40}/_{50}$

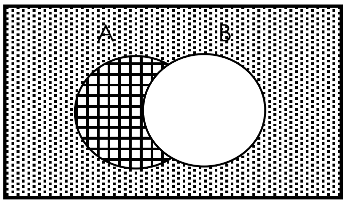
B : member likes 'COFFEE' $P(B) = {}^{20}/_{50}$

$A \cap B$: member likes 'BOTH' $P(A \cap B) = {}^{15}/_{50}$

E $\equiv$ member likes tea if its found that he does not like coffee

E $\equiv$ A | B'

$$P(E) = P(A | B')$$



$$= \frac{P(A \cap B')}{P(B')}$$

$$= \frac{P(A) - P(A \cap B)}{P(B')}$$

$$= \frac{{}^{40}/_{50} - {}^{15}/_{50}}{1 - {}^{20}/_{50}}$$

$$= \frac{{}^{25}/_{50}}{{}^{30}/_{50}} \qquad = {}^{5}/_{6}$$

17. a pair of dice is rolled . If the sum of the numbers appeared is 8 , find the probability that one die shows number 3

SOLUTION :

A : sum of numbers on the two dice is 8

$\equiv$ (2,6) , (3,5) , (4,4) , (5,3) , (6,2) $\qquad$ n(A) = 5 $\qquad \therefore$ P(A) = 5/36

B : one die show number 3

$\equiv$ (1,3) , (2,3) , (3,1) , (3,2) , (3,4) , (3,5), (3,6) , (4,3) , (5,3) , (6,3)

n(B) = 10 $\qquad \therefore$ P(B) = 10/36

$A \cap B \equiv$ (3,5) , (5,3) $\qquad$ $n(A \cap B) = 2 \quad \therefore P(A \cap B) = 2/36$

E $\equiv$ one die shows 3 if the sum of the numbers appeared is 8

E $\equiv$ B | A

$$P(E) = P(B | A)$$

$$P(E) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{{}^{2}/_{36}}{{}^{5}/_{36}} \qquad = 2 / 5$$

18. two fair dice are thrown . Find the probability that the sum of the points is at least 10 given that it exceeds 7

SOLUTION :

A : sum of points on the two dice is at least 10

(i.e. sum is 10 , 11 OR 12)

$\equiv$ (4,6) , (5,5) , (6,4) , (5,6) , (6,5) , (6,6) n(A) = 6 $\therefore$ P(A) = 6/36

B : sum of points exceeds 7

(i.e. sum is 8 , 9 , 10 , 11 OR 12)

$\equiv$ (2,6) , (3,5) , (4,4) , (5,3) , (6,2) ,

(3,6) , (4,5) , (5,4) , (6,3) ,

(4,6) , (5,5) , (6,4) ,

(5,6) , (6,5) ,

(6,6)

n(B) = 15 $\therefore$ P(B) = 15/36

$A \cap B$ $\equiv$ (4,6) , (5,5) , (6,4) , (5,6) , (6,5) , (6,6) $n(A \cap B)$ = 2 $\therefore$ $P(A \cap B)$= 6/36

E $\equiv$ sum of the points is at least 10 given that it exceeds 7

E $\equiv$ A | B

$$P(E) = P(A \mid B)$$

$$P(E) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{6/36}{15/36} \qquad = 2 / 5$$

19. 2 integers are selected at random from integers 1 to 11 . If the sum of the integers is even , find the probability that both the numbers are odd . ans : 15⁄25

SOLUTION :

exp : 2 integers are selected at random from integers 1 to 11 $n(S) = 11C2 = 55$

A : both the numbers are odd

$n(A) = {}^6C_2. = 15$; $P(A) = 15/55$

B : sum of integers is even

In that case either both the integers need to be ODD or both need to be even

$n(B) = {}^6C_2 + {}^5C_2. = 15 + 10 = 25$; $P(B) = 25/55$

$A \cap B$: sum of integers is even and both the numbers are ODD

$n(A \cap B) = {}^6C_2. = 15$; $P(A \cap B) = 15/55$

E : both the numbers are odd if the sum of integers is even

$E \equiv A | B$

$P(E) = P(A | B)$

$= \dfrac{P(A \cap B)}{P(B)} = \dfrac{15}{25}$

20. if $P(A) = 1/3$; $P(B) = 2/5$; $P(A \cup B) = 8/15$, then find a) $P(A | B)$ b) $P(B | A)$

SOLUTION :

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= \dfrac{1}{3} + \dfrac{2}{5} - \dfrac{8}{15}$

$= \dfrac{5 + 6 - 8}{15}$

$= \dfrac{3}{15}$

$= \dfrac{1}{5}$

$P(A | B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{1/5}{2/5} = 1/2$

$P(B | A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{1/5}{1/3} = 3/5$

21. IF A and B are two events of the sample space S , such that

$P(A \cup B) = 5/6$; $P(A \cap B) = 1/3$; $P(B) = 1/3$. Find a) $P(A' \cap B')$ b) $P(B' | A)$

SOLUTION :

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\frac{5}{6} = P(A) + \frac{1}{3} - \frac{1}{3}$$

$$P(A) = 5/6$$

$$P(A' \cap B') = 1 - P(A \cup B) = 1 - 5/6 = 1/6$$

$$P(B' | A) = \frac{P(A \cap B')}{P(A)} = \frac{P(A) - P(A \cap B)}{P(A)}$$

$$= \frac{\frac{5}{6} - \frac{1}{3}}{\frac{5}{6}} = \frac{\frac{3}{6}}{\frac{5}{6}} = 3/5$$